IPPP/08/29
DCPT/08/58

arXiv:0906.5065v1 [hep-ph] 29 Jun 2009

Constraining New Physics in $B^0 \to \pi^+\pi^-$ with Reparametrization Invariance and QCD Factorization

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Abstract

Usually, $B^0 \to \pi^+\pi^-$ decays are expressed in terms of weak amplitudes explicitly dependent on the CKM weak phase α or γ. In this letter, we show that the weak amplitudes can be rewritten such that a manifest dependence on β emerges instead. Based on this, we constrain new-physics contributions to the CP-violating phase ϕ_d in B^0–$\bar{B}^0$ mixing. Further, we apply reparametrization invariance and use QCD factorization predictions to investigate the bounds on an additional new-physics amplitude in $B^0 \to \pi^+\pi^-$.

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One of the greatest successes of the B factories BaBar and Belle is the precise determination of the CP-violating phase ϕ_d in B mixing [1]. In the Standard Model (SM), and using the Wolfenstein parametrization of the CKM matrix, ϕ_d is related to β, one of the angles of the unitarity triangle, as $\phi_d = 2\beta$. As B mixing is a loop process, the experimentally determined angle ϕ_d might in fact not equal 2β, but be polluted by the effects of new-physics (NP) particles propagating in loops, thereby contributing an additional CP violating phase, see for instance Ref. [2].[1] It is therefore of considerable interest to study any methods by which one can constrain an additional NP contribution to ϕ_d. In this letter we shall show that the process $B^0 \to \pi^+\pi^-$ can be used to this effect.

The set of neutral and charged $B \to \pi\pi$ decays has been extensively studied as a means of determining the angle α (or γ) of the unitarity triangle. The lack of a theoretically clean calculation of the strong amplitudes and phases involved can be overcome by exploiting isospin symmetry, see Ref. [4], commonly referred to as the Gronau-London method. It involves relating the various experimental observables (branching ratios and CP asymmetries) in $B \to \pi\pi$ to extract both the hadronic amplitudes determining these decays and the weak phase α. As an alternative to isospin, and in order to avoid $B^0 \to \pi^0\pi^0$ decays, the use of U-spin has been explored in Refs. [5] to extract γ from $B^0 \to \pi^+\pi^-$ and the U-spin related decay $B_s \to K^+K^-$. In a conceptionally different approach the relevant strong amplitudes are calculated (as opposed to extracted from experiment), using QCD factorization (QCDF) [6, 7, 8, 9] or effective field theory methods (SCET) [10]. The advantage here is that less experimental input is needed, the disadvantage that the calculation is performed in a limit of QCD where the b quark is assumed to be very heavy. In any case, all these analyses put the emphasis on constraining the angles γ or α.

In this letter, we show that it is possible to express the decay amplitude in terms of ϕ_d and β, without any explicit reference to the angles α or γ. In the SM, the resulting expression allows the extraction of the relevant hadronic parameters from $B^0 \to \pi^+\pi^-$ data alone, which can be compared to the theoretical calculation in QCDF. Beyond the SM, we study the possible presence of NP in this decay, which might contribute through B^0–$\bar{B}^0$ mixing via a NP contribution to ϕ_d or through an additional NP amplitude.

We begin with a reminder of the parametrization used to extract γ. The amplitude for $\bar{B}^0 \to \pi^+\pi^-$ is given in the form:

$$\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) = \lambda_c A_c + \lambda_u A_u \,, \tag{1}$$

where $\lambda_q = V_{qd}^* V_{qb}$, and A_c, A_u are strong amplitudes. A_u is dominated by tree diagrams, whereas the only contributions to A_c are from penguin diagrams. The corresponding time-dependent CP asymmetry is given by:

$$\begin{aligned} A_\pm(t) &= \frac{\Gamma(B^0(t) \to \pi^+\pi^-) - \Gamma(\bar{B}^0(t) \to \pi^+\pi^-)}{\Gamma(B^0(t) \to \pi^+\pi^-) + \Gamma(\bar{B}^0(t) \to \pi^+\pi^-)} \\ &= C_\pm \cos(\Delta m t) - S_\pm \sin(\Delta m t) \,. \end{aligned} \tag{2}$$

The experimental observables $C_\pm$ and $S_\pm$ can be expressed in terms of $\lambda_\pm$:

$$S_\pm = \frac{2\,\mathrm{Im}(\lambda_\pm)}{1+|\lambda_\pm|^2}\,, \qquad C_\pm = \frac{1-|\lambda_\pm|^2}{1+|\lambda_\pm|^2}\,, \tag{3}$$

[1] NP at tree level this is highly disfavoured as its impact on B mixing would exceed the observed effects by far – unless the mass scales involved are in the $\sim 10\,\mathrm{TeV}$ range, see Ref. [3].

Parameter	Value	Source
λ	$0.2257^{+0.0009}_{-0.0010}$	PDG [11]
$\lvert V_{cb}\rvert$	$(41.2 \pm 1.1) \times 10^{-3}$	PDG [11]
$\lvert V_{ub}\rvert$	$(3.93 \pm 0.36) \times 10^{-3}$	PDG [11]
$\beta_{b\to ccs}$	$(21.1 \pm 0.9)^\circ$	HFAG [1]
$\beta_{\rm tree}$	$(23.9 \pm 3.3)^\circ$	this paper, Eq. (16)
γ	$(77^{+30}_{-32})^\circ$	PDG [11]
R_b	0.412 ± 0.039	this paper, Eq. (6)
$\left\lvert \frac{V_{td}}{V_{ts}} \right\rvert$	0.214 ± 0.005	[12]
R_t	0.928 ± 0.024	this paper, Eq. (10)

Table 1: CKM parameters used in this letter.

where $\lambda_\pm$ is given by

$$\lambda_\pm = e^{-i\phi_d}\,\frac{\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-)}{\mathcal{A}(B^0 \to \pi^+\pi^-)}\,. \tag{4}$$

Parametrizing the amplitudes as in Eq. (1), we have

$$\lambda_\pm = e^{-i\phi_d}\,\frac{e^{-i\gamma} - re^{i\delta}}{e^{i\gamma} - re^{i\delta}} \tag{5}$$

with $re^{i\delta} = A_c/(A_u R_b)$, $\gamma = \arg(-\lambda_c/\lambda_u)$ and

$$R_b = \left|\frac{\lambda_u}{\lambda_c}\right| = \frac{1-\lambda^2/2}{\lambda}\,\frac{|V_{ub}|}{|V_{cb}|}\,. \tag{6}$$

Numerical values for these and other CKM-related quantities are collected in Tab. 1. The observables $S_\pm$ and $C_\pm$ are given by

$$S_\pm = \frac{\sin(\phi_d + 2\gamma) - 2r\sin(\phi_d+\gamma)\cos\delta + r^2\sin\phi_d}{1 - 2r\cos\gamma\cos\delta + r^2}\,, \tag{7}$$

$$C_\pm = -\frac{2r\sin\gamma\sin\delta}{1 - 2r\cos\gamma\cos\delta + r^2}\,. \tag{8}$$

In the absence of penguin contributions, $r = 0$ and the determination of $\phi_d + 2\gamma$ would be completely analogous to that of ϕ_d from $B^0 \to J/\psi K_S$. Realistically, r is expected to be a small, but non-zero number, which makes the extraction of γ more challenging.

We now show how a different parametrization of the decay amplitude (1) replaces the explicit dependence of $\lambda_\pm$ on γ by one on β. Using $\beta = \arg(-\lambda_t/\lambda_c)$, one can trade the dependence on γ for one on β by exploiting the unitarity of the CKM matrix and exchanging λ_u for $-\lambda_c - \lambda_t$:

$$\begin{aligned}\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) &= \lambda_c B_c + \lambda_t B_t \\ &= \lambda_c (B_c - R_t e^{i\beta} B_t)\,,\end{aligned} \tag{9}$$

where $B_c = A_c - A_u$, $B_t = -A_u$ and

$$R_t = \left|\frac{\lambda_t}{\lambda_c}\right| = \frac{1}{\lambda}\frac{|V_{td}|}{|V_{ts}|}\left\{1 - \frac{1}{2}\left(1 - 2R_b\cos\gamma\right)\lambda^2 + O(\lambda^4)\right\}. \tag{10}$$

Note that B_c and B_t are both dominated by tree-level decays as they both contain A_u.

With this parametrization of the decay amplitude, $\lambda_\pm$ becomes

$$\lambda_\pm = e^{-i\phi_d}\left(\frac{1 - R_t R_{tc} e^{i\beta}}{1 - R_t R_{tc} e^{-i\beta}}\right) \tag{11}$$

$$= e^{-i\phi_d}\left(\frac{1 - d e^{i\theta_d} e^{i\beta}}{1 - d e^{i\theta_d} e^{-i\beta}}\right), \tag{12}$$

where $R_{tc} = B_t/B_c$ and $d = |R_t R_{tc}|$, $\theta_d = \arg(R_t R_{tc})$. Note that unlike r, d is not suppressed, but expected to be of order 1 (as R_t is also close to 1). The CP-violating observables in (3) now read

$$S_\pm = \frac{d^2\sin(2\beta - \phi_d) + 2d\cos\theta_d\sin(\phi_d - \beta) - \sin(\phi_d)}{d^2 - 2d\cos\beta\cos\theta_d + 1}, \tag{13}$$

$$C_\pm = -\frac{2d\sin\beta\sin\theta_d}{d^2 - 2d\cos\beta\cos\theta_d + 1}. \tag{14}$$

Obviously (13), (14) are not independent of (7), (8), but related by the unitarity constraint

$$R_t e^{i\beta} + R_b e^{-i\gamma} - 1 = 0\,. \tag{15}$$

The advantage of expressing $S_\pm$ and $C_\pm$ in terms of β instead of γ is that, at least in the SM, there is now only one manifest weak phase. This implies that, with R_t determined from B mixing, both d and θ_d can be extracted from experiment and compared to theoretical calculations, for example QCDF. This is independent of any information from the decay $B^0 \to \pi^0\pi^0$ whose branching ratio continues to be difficult to understand in the framework of QCDF or SCET.

The most accurate measurement of ϕ_d is via mixing in B^0 decays to CP eigenstates of charmonium. The CP asymmetry averaged over these channels provides a direct measurement of $\sin\phi_d = 0.673 \pm 0.023$, so that in the SM $\beta = (21.1 \pm 0.9)^\circ$ [1][2]. It is also possible to derive β from tree-process measurements only, based on γ and $|V_{ub}|$. Taking γ and $|V_{ub}|$ from Ref. [11], see Tab. 1, we find $\beta_{\rm tree}$ using

$$\sin\beta_{\rm tree} = \frac{R_b\sin\gamma}{\sqrt{1 - 2R_b\cos\gamma + R_b^2}}\,, \qquad \cos\beta_{\rm tree} = \frac{1 - R_b\cos\gamma}{\sqrt{1 - 2R_b\cos\gamma + R_b^2}}\,, \tag{16}$$

which results in $\beta_{\rm tree} = (23.9^{+3.4}_{-3.2})^\circ$ (in the following analysis we use $\beta_{\rm tree} = (23.9 \pm 3.3)^\circ$). Both values of β are compatible with each other, but we will use the latter one to obtain constraints on a NP contribution to ϕ_d.

[2] There is an ambiguity in this result, as $\beta = (68.9 \pm 1.0)^\circ$ is also a solution. However, this is excluded at the 95% confidence level by a Dalitz plot analysis of $B^0 \to \bar{D}^0 h^0$ where $h^0 = \pi^0, \omega, \eta$ [13], and by a time-dependent angular analysis of $B^0 \to J/\psi K^{*0}$ [14].

Experiment	$S_\pm$	$C_\pm$
BaBar [15]	$-0.68 \pm 0.10 \pm 0.03$	$-0.25 \pm 0.08 \pm 0.02$
Belle [16]	$-0.61 \pm 0.10 \pm 0.04$	$-0.55 \pm 0.08 \pm 0.05$
HFAG [1]	-0.65 ± 0.07	-0.38 ± 0.06

Table 2: Experimental results for $S_\pm$, $C_\pm$ from BaBar and Belle and the HFAG average.

	$\beta_{b\to ccs} = (21.1 \pm 0.9)^\circ$		$\beta_{\text{tree}} = (23.9 \pm 3.3)^\circ$	
	d	θ_d	d	θ_d
BaBar	0.790 ± 0.031	0.068 ± 0.025	0.775 ± 0.037	0.075 ± 0.028
Belle	0.803 ± 0.033	0.158 ± 0.041	0.789 ± 0.040	0.174 ± 0.049
HFAG	0.796 ± 0.021	0.104 ± 0.020	0.782 ± 0.027	0.115 ± 0.025
QCDF	$0.825^{+0.034}_{-0.052}$	$-0.021^{+0.043}_{-0.042}$	$0.825^{+0.034}_{-0.052}$	$-0.021^{+0.043}_{-0.042}$

Table 3: Comparison of d and θ_d derived from experimental results to QCDF predictions for $\beta_{b\to ccs}$ from $b \to ccs$ transitions and β_{tree} derived from tree decays.

From the experimental data collected in Tab. 2, we find the values of d and θ_d given in Tab. 3. The high quality of the experimental results leads to small uncertainties on d, typically 5%, and moderate uncertainties on θ_d. As expected, d is of order 1, while θ_d is rather small.[3]

How do these results compare to theoretical predictions? As mentioned earlier, QCDF provides us with a framework for computing the individual amplitudes contributing to $B^0 \to \pi^+\pi^-$, and the relative phases between them. Complete results for both tree and penguin contributions are available to NLO accuracy [7]. The result for A_c/A_u is:[4]

$$\frac{A_c}{A_u} = -0.122^{+0.033}_{-0.063} - 0.024^{+0.047}_{-0.048} i \,. \tag{17}$$

Therefore using $B_c = A_c - A_u$, $B_t = -A_u$, we find

$$R_{tc} = \frac{B_t}{B_c} = 0.891^{+0.026}_{-0.050} - 0.019^{+0.037}_{-0.038} i \,. \tag{18}$$

As R_t is, by definition, a positive number, θ_d is given by the phase of R_{tc}, as seen in Tab. 3. The theoretical prediction is ca. 3σ away from the experimental (HFAG) result and has the opposite sign. From Eq. (14) it is clear that θ_d is mainly constrained by $C_\pm$.

[3] Actually, there is a discrete ambiguity in the determination of d and θ_d which yields a second solution $d \sim 0.3$ and $\theta_d \sim 1$. We discard this solution as d is the ratio of two tree-dominated amplitudes and hence expected to be close to 1; $d \sim 0.3$ would imply a massive NP amplitude which can not be generated in any NP models we are aware of.

[4] Recently the NNLO calculation of the tree amplitude has been completed in Ref. [9]. We do not use this result in our analysis, as we also require the penguin amplitude at NNLO. Nevertheless, we have checked numerically that using this value would alter A_c/A_u by 4 to 5%.

While Tab. 2 clearly shows that the results from BaBar and Belle are not completely in agreement, they *do* agree on the sign and require a positive θ_d. This situation is similar to that for $B \to K\pi$ decays, where the sign of the observed direct CP violation is difficult to reconcile with QCDF predictions. The source of the discrepancy is independent of any NP contributions to $\Delta m_{d,s}$, from which R_t is determined, as θ_d is independent of R_t. It is also unrelated to any NP contributions to the B mixing phase ϕ_d, as using the tree-level determination of β, $\beta_{\rm tree}$, for the explicit β dependence renders the discrepancy even slightly worse (right-hand side of Tab. 3). An explanation in terms of NP could be that an additional NP amplitude contributes to $B^0 \to \pi^+\pi^-$, and we explore this possibility below. Otherwise, the discrepancy may be related to neglected higher order terms in QCDF – either from radiative corrections or terms suppressed by inverse powers of the b quark mass. As for extreme values of the QCDF input parameters a positive θ_d *is* possible, it would be interesting to see what choice of these parameters yields positive θ_d and whether this choice agrees with the scenarios advocated in Refs. [7, 8, 9] to reconcile the QCDF predictions for $B(B^0 \to \pi^0\pi^0)$ with experimental data.

In order to calculate d from QCDF we also need R_t. This can be determined from the ratio of the mass differences in the B_s and B_d systems. CDF has made a very clean measurement of Δm_s [17], which can be turned into a value for R_t using lattice information on the relevant hadronic parameters [12] and Eq. (10). The result is given in Tab. 1. This value of R_t allows for a limited contribution of NP to the mass differences in the $B_{d,s}$ systems: from the unitarity relation (15), using R_t and β as input, one finds $R_b \approx 0.36$ and $\gamma \approx 68^\circ$, in good agreement with the values quoted in Tab. 1.[5] Combining the QCDF calculation with this value for R_t, we find that the QCDF prediction agrees with the experimental results within 1σ. The experimental results and QCDF predictions for d and θ_d are shown in Fig. 1. The analogous QCDF predictions needed in the "standard" parametrization of $S_\pm$, Eq. (7), and $C_\pm$, Eq. (8), can be found by dividing (17) by R_b:

$$r = 0.30^{+0.15}_{-0.09}, \qquad \delta + \pi = 0.194^{+0.384}_{-0.382}\,. \tag{19}$$

Note the marked difference in the relative errors for (d, θ_d) and (r, δ), which is an important advantage to the parametrization (9) of the $B^0 \to \pi^+\pi^-$ amplitude and will reduce the uncertainties in the constraints on a NP amplitude contribution to this decay.

Let us now discuss constraints on $\phi_d - 2\beta$, based on the QCDF results in Tab. 3. Assuming there is no NP amplitude contributing to the decay, one can obtain β using ϕ_d and d, θ_d from QCDF as input. This results in $\beta = 19.9^\circ$ with a minimum χ^2 of 63. Leaving θ_d as fit parameter instead, we find $\beta = (16.4 \pm 6.5)^\circ$ which shows that the above discrepancy in θ_d from QCDF and from experiment is largely irrelevant for NP contributions to ϕ_d. Both values are consistent with $\phi_d/2 = (21.1 \pm 0.9)^\circ$, and and also with $\beta = (23.9 \pm 3.3)^\circ$ determined from γ and $|V_{ub}|$. However, it is still interesting to note the two results for $\phi_d/2 - \beta$ are of opposite sign. If there was indeed a NP contribution to ϕ_d, we would expect to see common trend in the sign of $\phi_d/2 - \beta$ from various determination. As this does not seem to be the case, we take this as an indication that any NP contribution to ϕ_d is indeed small.

Given this result, the decay $B^0 \to \pi^+\pi^-$ seems then a very suitable place to constrain any NP contributions to the decay amplitude. Such contributions are possible in a variety

[5]The agreement is not perfect for R_b. If, however, one uses the result for $|V_{ub}| \approx 3.5 \times 10^{-3}$ from exlusive decays, instead of the larger PDG value, $R_b \approx 0.36$ and the agreement is near perfect.

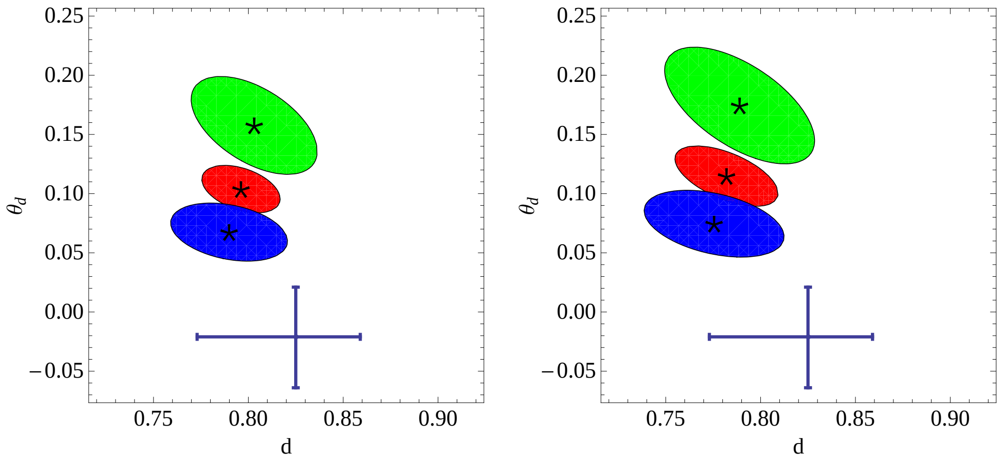

Figure 1: The allowed experimental region for d and θ_d are plotted for the case $\beta = (21.1 \pm 0.9)^\circ$ on the left and $\beta = (23.9 \pm 3.3)^\circ$ on the right. The blue (lower) region corresponds to the results for $S_\pm$ and $C_\pm$ of BaBar, the green (upper) region corresponds to those of Belle and the red (central) region corresponds to the average given by HFAG as in Tab. 2. The blue point with error bars represents the QCDF prediction.

of NP models, e.g. the 2-Higgs doublet model [18] or R-parity violating SUSY [19]. We therefore introduce the (complex) NP amplitude $A_{\rm NP}$ and a new weak phase $\delta_{\rm NP}$, such that

$$\lambda_\pm = e^{-i\phi_d} \left(\frac{1 - d e^{i\theta_d} e^{i\beta} + A_{\rm NP} e^{i\delta_{\rm NP}}}{1 - d e^{i\theta_d} e^{-i\beta} + A_{\rm NP} e^{-i\delta_{\rm NP}}} \right). \tag{20}$$

Here d and θ_d are SM quantities, given by the QCDF values stated earlier, in Tab. 3. The above expression for $\lambda_\pm$ is actually not suitable to constrain $A_{\rm NP}$ and $\delta_{\rm NP}$: in Ref. [20], it was shown that a given amplitude, with an arbitrary number of distinct weak phases, can always be expressed in terms of any two weak phases. Using this so-called reparametrization invariance, $\lambda_\pm$ can be expressed in terms of the two weak phases ϕ_d and β:[6]

$$\lambda_\pm = e^{-i\phi_d} \left(\frac{1 - d' e^{i\theta'_d} e^{i\beta}}{1 - d' e^{i\theta'_d} e^{-i\beta}} \right), \tag{21}$$

where

$$d' e^{i\theta'_d} = \frac{d e^{i\theta_d} - A_{\rm NP} \sin(\delta_{\rm NP}) / \sin\beta}{1 + A_{\rm NP} \sin(\beta - \delta_{\rm NP}) / \sin\beta}. \tag{22}$$

Note that $d' e^{i\theta'_d} \to d' e^{i\theta'_d}$ under a CP transformation, so θ'_d is indeed a strong phase. In Eq. (21), ϕ_d is, by definition, the phase measured in B mixing, i.e. from $b \to ccs$ transitions, while β is obtained from tree-level processes, i.e. $\phi_d = (42.2 \pm 1.8)^\circ$ and $\beta = (23.9 \pm 3.3)^\circ$. The values for d' and θ'_d are obviously identical to the experimental results for d and θ_d in Tab. 3. The resulting constraints on $A_{\rm NP}$ and $\delta_{\rm NP}$ are shown in Fig. 2. Depending on the value of $\delta_{\rm NP}$, large NP contributions $|A_{\rm NP}|$ are possible. Note that these results are mainly due to the discrepancy between θ'_d from experiment and θ_d from QCDF. In particular, even for $\delta_{\rm NP} = 0$ a non-zero $|A_{\rm NP}| \approx 0.1$ is needed. Also note

[6] Eq. (21) was already obtained in Ref. [20], as (48).

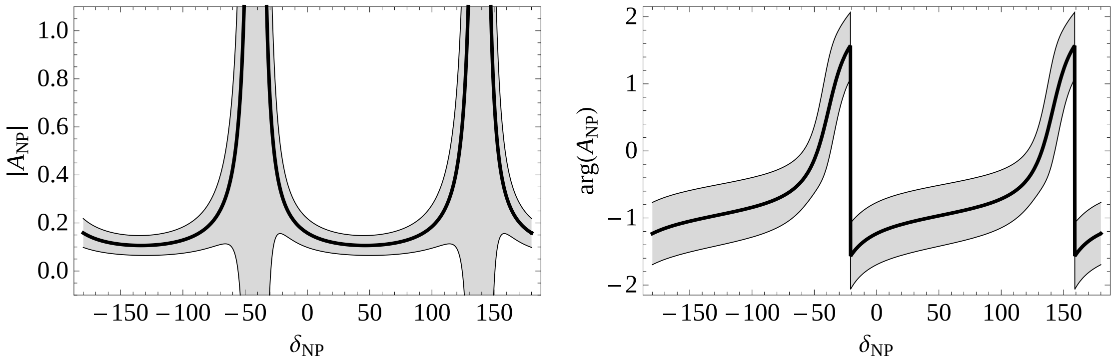

Figure 2: $|A_{\rm NP}|$ and $\arg(A_{\rm NP})$ plotted against the weak phase of the new physics amplitude, $\delta_{\rm NP}$. The thick black lines represent the results found using the experimental average value of $\sin(\phi_d)$, the grey regions represent the uncertainty.

that the allowed valued for $|A_{\rm NP}|$ can be as large, or even larger, than the SM (QCDF) prediction for d. Although it would be interesting to interpret the NP amplitude in terms of the NP models mentioned, making use of constraints from other processes, this is beyond the scope of this letter.

In summary, we have investigated the effect of NP on the decay $B^0 \to \pi^+\pi^-$. We first analyzed the CP asymmetries in the SM, using a particularly convenient parametrization of the decay amplitude which depends on only one weak phase, β. We compared the experimental results of the relevant hadronic quantities with those from theoretical calculations, using QCDF. We found that while the size of the strong amplitude is consistent, its predicted phase θ_d deviates from experiment by $\sim 3\sigma$. It would be interesting to study whether the choice of QCDF input parameters minimizing this discrepancy is the same needed to reduce that between the predicted and observed branching ratio of $B^0 \to \pi^0\pi^0$. We then analyzed the implications for NP contributions to the decay. Under the assumption that NP modifies only the phase in B mixing, the discrepancy in the strong phase was largely irrelevant and the NP contributions to the mixing phase found to be consistent with 0. We then allowed for a NP amplitude contributing to the decay amplitude of $B^0 \to \pi^+\pi^-$ and analyzed the CP asymmetries exploiting the constraints imposed by reparametrization invariance. Here we found scope for large NP contributions, which are driven by the discrepancy between θ_d from experiment and from QCDF. An analysis of this NP amplitude within specific models is beyond the scope of this letter.

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